# A CP-based Automatic Tool for Instantiating Truncated Differential Characteristics

François Delobel Patrick Derbez **Arthur Gontier** Loïc Rouquette Christine Solnon

March 30, 2026

# Cryptography

Communicate a secret:

- Confidentiality
- Integrity
- Authentication
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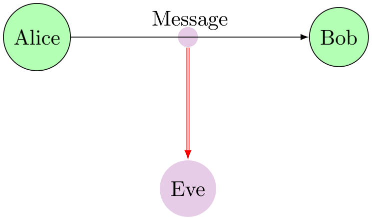

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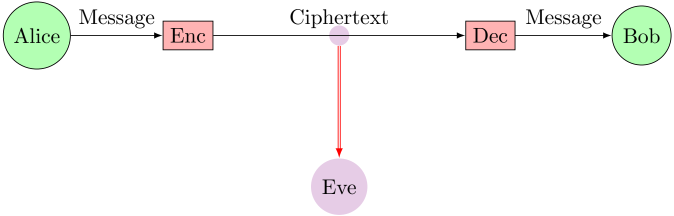

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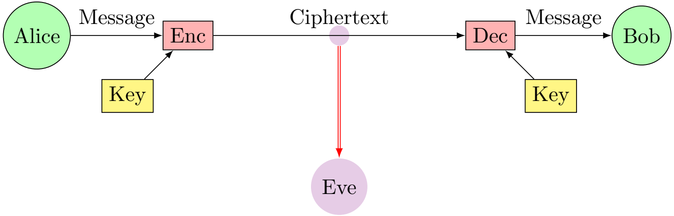


# Symmetric ciphers

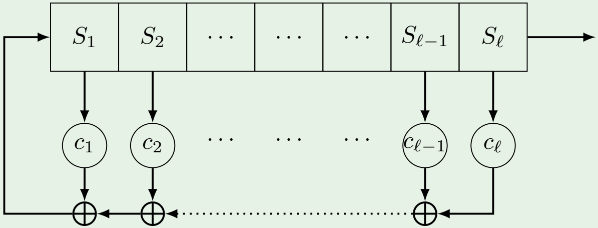
An invertible function $E$:

$$E : key \times message \rightarrow ciphertext$$

An iterated round function $f$:

$$E = f(f(\ldots f(f(key, message))\ldots))$$

### Stream cipher example: Linear Feedback Shift Register (LFSR)



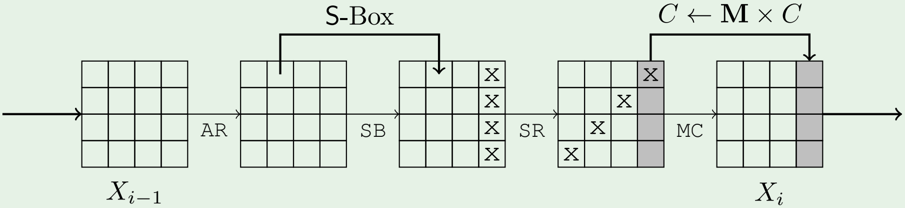

Some stream ciphers: A5/1 (GSM 1987), RC4 (Wifi 1987), E0 (Bluetooth 1999), **Trivium** (2004),...

# Symmetric ciphers

Properties of a resistant cipher:

- Diffusion (permutation, XOR,. . . )
- Confusion (Substitution Box)

**Block cipher example: Advanced Encryption Standard (AES)**




Some block ciphers families: **Feistel networks**, Substitution permutation networks, ARX,. . .

# Distinguishers


Distinguish a cipher from a random message

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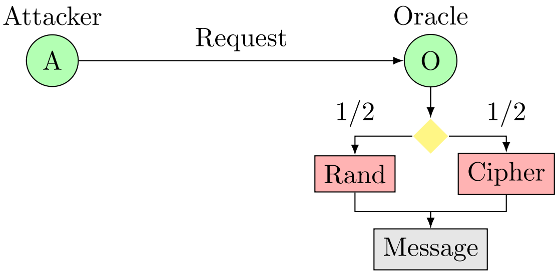

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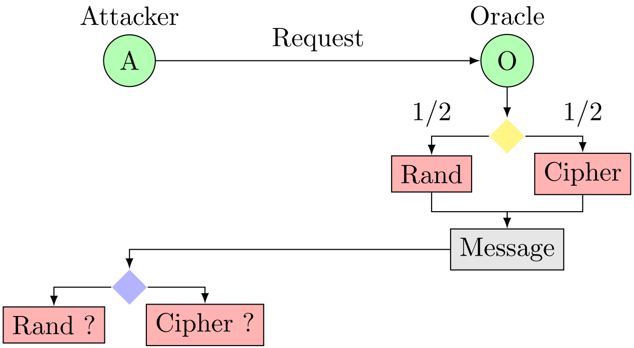
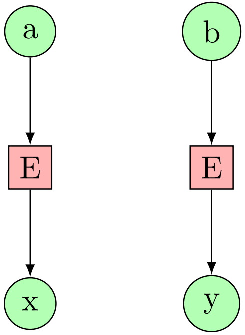


# Differentials [BS93, BS90]

## Differential characteristics

- Difference: $a \oplus b = \delta$
- Probability: $P(E(x) = E(x \oplus \delta_{in}) \oplus \delta_{out})$



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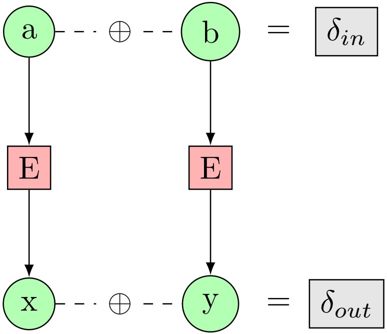

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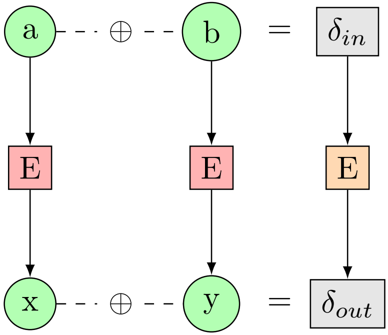

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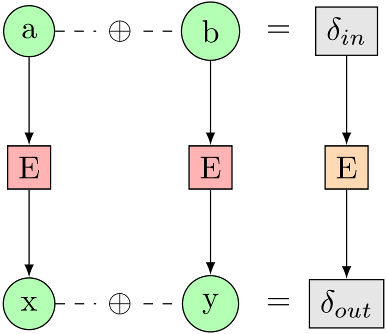
## Truncated characteristics [Knu94]

$$\Delta x_i = \begin{cases} 0 & \text{if} \quad \delta x_i = 0 \\ 1 & \text{if} \quad \delta x_i \in [\![1, 2^n - 1]\!] \end{cases}$$



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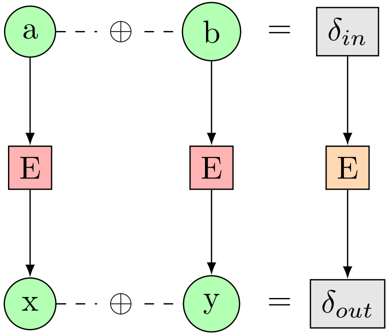
Two steps method [BN10, FJP13, GLMS20]

### Solving methods

Branch and bound, dynamic programming, generic solvers, ...



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- Conflict Driven Clause Learning

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### Solving CP model

- Branch and bound
- Filtering algorithms

↪ Choco

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- Each cipher must be resistant to differentials

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**Automated tools for differentials ?**

- YAARX [Leu12] dedicated algorithms (ARX)
- CryptoSMT [Köl] SMT models
- TAGADA [LDLS21] DAG to MiniZinc models
- CASCADA [RR22] SMT models
- CLAASP [BGG+23] DAG to MiniZinc models

# Automatic differential cryptanalysis

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**Problem**

- Each cipher must be resistant to differentials

**A tool for truncated differentials: Tagada**

- **Input:** cipher DAG and optional bound
- **Output:** truncated solutions

**Generate** a MiniZinc model and solve it (Solver: PicatSAT)

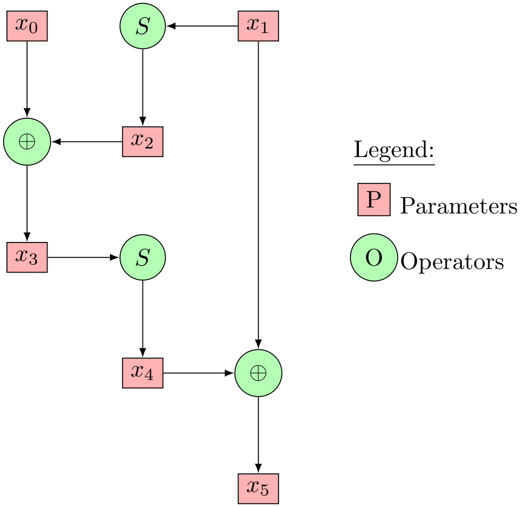
# Unified description of ciphers

## Directed Acyclic Graph (DAG)

- **Parameters:** variables or constants
- **Operators:** cipher operators

## Example (S operator)

- Domain: $[\![0, 256]\!]$
- Co-Domain: $[\![0, 256]\!]$
- Function:
  - type: S-Box
  - lookup table $[170, 22, 3, \ldots]$



# Second step with CP (Choco)

**Contribution**

- **Input:** cipher DAG, truncated solutions and optional bound
- **Output:** differentials of best probability

**Generate** a CP model and solve it (Solver: Choco)

$\hookrightarrow$ **How to model all the operators ?** (S-Boxes, XORs, LFSRs, Galois Fields operations,. . . )

# S-Box and Differential Distribution Table (DDT)

### Computing DDT

$DDT(\delta_{in}, \delta_{out}) = \#\{X | S(X) \oplus S(X \oplus \delta_{in}) = \delta_{out}\}$

### Modeling DDT with table constraint

- List of tuples: $tuple(\delta_{in}, \delta_{out}, Prob)$
- **Filtering:** efficient data structure to retain always one valid tuple

| DDT | 0 | 1 | 2 | 3 | 4 | ... |
|---|---|---|---|---|---|---|
| 0 | **64** | 0 | 0 | 0 | 0 | |
| 1 | 0 | 0 | 0 | 6 | 0 | |
| 2 | 0 | 0 | 0 | 8 | 0 | ... |
| 3 | 14 | 4 | 2 | 2 | 10 | |
| 4 | 0 | 0 | 0 | 6 | 0 | |
| ⋮ | | | ⋮ | | | ⋱ |

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| ⋮ | | | ⋮ | | | ⋱ |

$$tuple(0, 0, 1)$$
$$tuple(3, 0, \frac{14}{64})$$
$$tuple(3, 1, \frac{4}{64})$$
$$\ldots$$

# XOR filtering algorithm

## Previous works

- Table constraint
- Dedicated algorithm

## Filtering quality

- The set is unlikely to filter values.

↪ Filtering condition: $\#D_a \times \#D_b \leq \#D_c$

**Algorithm 1:** 3-variable XOR filtering

```
Input: IntVar a, IntVar b, IntVar c (target)
set ← ∅;
// Loop through possible values
for all v1 ∈ D_a do
    for all v2 ∈ D_b do
        set ← set ∪ {v1 ⊕ v2};
D_c ← D_c ∩ set;
```

# Other filtering algorithms

| Non-linear Operators | | |
|---|---|---|
| Operator | Name | Constraint |
| $DDT$ | Differential Distribution Table | Table |

| Linear Operators | | |
|---|---|---|
| $\oplus$ | N-ary Bitwise XOR | Custom |
| $\otimes_K$ | Galois Field Multiplication with Constant | |
| LFSR | Linear Feedback Shift Register | |
| $\ll$ or $\gg$ | Left (Right) Shift | |
| $\lll$ or $\ggg$ | Left (Right) Circular Shift | |
| $\odot_K$ | Galois Field Matrix Multiplication with Constant Matrix | Decomposition to $\otimes_K$ and $\oplus$ |
| $=$ | Equal | Native |
| $\&_K$ | Bitwise AND with Constant | Table |
| $\Vert_K$ | Bitwise OR with Constant | |
| $AB \rightarrow (A, B)$ | Split | |
| $(A, B) \rightarrow AB$ | Concat | |
| T | Linear Lookup Table | |

# Optimisations

## Optimizations

- **Simplification:** Remove inactive S-Boxes
- **Heuristic:** Start search near S-Boxes
- **Solving:** Parallel competitive models
- **Solving:** Two steps together

**Algorithm 2:** Twostep

```
List1 ← Step1-enum(LB) ;
List2 ← Step2-parallel(List1) ;
```

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**Algorithm 5:** Twostep

```
S1, UB ← Step1-opt() ;
while LB < UB do
    S2, LB ← Step2(S1, LB) ;
    if LB < UB then
        S1 ← Step1-next(UB) ;
        if S1 is null then
            S1, UB ← Step1-opt(UB)
```

# Contribution

**Tagada two steps results:**

- Reproduce all these results within a day

| Cipher | Max R | Proba | Ref |
|---|---|---|---|
| Midori-64 | 16 | $2^{-16}$ | [Gér18] |
| Midori-128 | 20 | $2^{-40}$ | |
| Warp | 41 | $2^{-40}$ | [TB22] |
| Twine-80 | 18 | $2^{-64}$ | [SMS+20] |
| Twine-128 | 16 | $2^{-52}$ | |
| Skinny-64-TK1 | 11 | $2^{-64}$ | [DDH+21] |
| Skinny-128-TK1 | 11 | $2^{-74}$ | |
| AES-128 | 5 | $2^{-105}$ | [GLMS20] |
| AES-192 | 9 | $2^{-146}$ | |
| AES-256 | 14 | $2^{-146}$ | |
| Rijndael-128-160 | 7 | $2^{-120}$ | [RGMS22] |
| Rijndael-128-224 | 12 | $2^{-212}$ | |
| Rijndael-160-128 | 4 | $2^{-112}$ | |
| Rijndael-160-160 | 6 | $2^{-138}$ | |

| Cipher | Max R | Proba | Ref |
|---|---|---|---|
| Rijndael-160-192 | 8 | $2^{-141}$ | [RGMS22] |
| Rijndael-160-224 | 9 | $2^{-190}$ | |
| Rijndael-160-256 | 11 | $2^{-204}$ | |
| Rijndael-192-128 | 3 | $2^{-54}$ | |
| Rijndael-192-160 | 5 | $2^{-118}$ | |
| Rijndael-192-192 | 7 | $2^{-153}$ | |
| Rijndael-192-224 | 8 | $2^{-205}$ | |
| Rijndael-192-256 | 9 | $2^{-179}$ | |
| Rijndael-224-128 | 3 | $2^{-54}$ | |
| Rijndael-224-160 | 4 | $2^{-122}$ | |
| Rijndael-224-192 | 5 | $2^{-124}$ | |
| Rijndael-224-224 | 7 | $2^{-196}$ | |
| Rijndael-224-256 | 8 | $2^{-182}$ | |
| Rijndael-256-128 | 3 | $2^{-54}$ | |
| Rijndael-256-160 | 4 | $2^{-130}$ | |
| Rijndael-256-192 | 5 | $2^{-148}$ | |
| Rijndael-256-224 | 4 | $2^{-115}$ | |
| Rijndael-256-256 | 6 | $2^{-128}$ | |

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- New filtering algorithms and optimizations
- A tool for fast differential cryptanalysis of word oriented ciphers

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  - ↪ Better cryptanalysis
- Other distinguishers
  - ↪ Boomerangs, impossible differentials,
- Other ciphers
  - ↪ Other solver ?
- Other tools using the DAG
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